Practical Illumination from Flames

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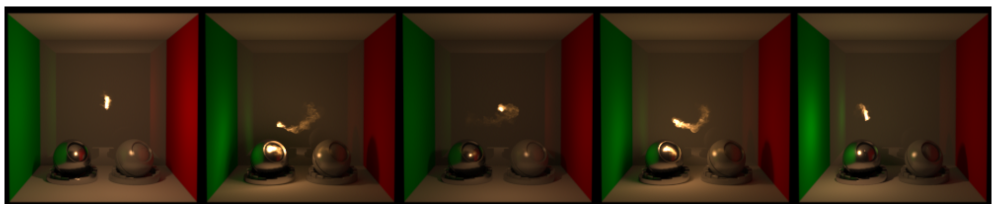

Figure 1. Flame in a Cornell box with specular, glossy, and diffuse objects. Our method provides a robust sampling for a wide range of materials.

Abstract

We present a method for creating and sampling volumetric light sources directly using the raw volumetric data under a Monte Carlo framework. Our algorithm does not require any preprocessing or baking, so it is compatible with progressive rendering. Since it does not use imposters, we can achieve high quality results with any mirror, glossy, or diffuse object. To be able to efficiently sample the volume emission, we use two strategies: light sampling (for the volume) and BRDF sampling (for the receiving object), and express them in the same domain so that we can apply multiple importance sampling (MIS) to compute the final result. Our technique focuses on the emission portion of volumetric rendering and is complementary to all other recent publications handling scattering effects.

1. Introduction

In the era of global illumination, it is only natural that emissive volumes interact with objects in the environment. These volumes need to be visible to ray tracing, and volumetric effects need to be accounted for, during the illumination pass. If the energy emitted from volumes is fairly low, they can be treated as any normal emissive object visible in reflections; however, if the emission becomes strong, we need to treat the volume as a light source with its own sampling strategy, in order to reduce variance.

In the past, such effects were achieved efficiently by converting the volume data into multiple point light sources (as described in Section 7.1 of Christensen's point-based lighting article [2004]), or by considering only the envelope of the volume

 ISSN 2331-7418

and simplifying it to a 2D surface sampling problem. With diffuse or highly glossy surfaces, those approaches can give good results, but for low gloss or mirror surfaces, these approximations can become apparent (Figure 8). In many cases, these techniques usually require time-consuming parameter tweaking and setup from the artists.

The main contribution of our paper is a way to importance sample a volume emission with two strategies: light sampling and BRDF sampling expressed in the same domain, so that we can use multiple importance sampling (MIS) [Veach and Guibas 1995] to further reduce variance. By combining specialized sampling with MIS, our technique is robust enough to account for any possible combination of BRDFs and emission types. Similar strategies have been employed before for infinite area lights by Pharr and Humphreys [2004]; we extend this method to volumetric lights and show how to handle the extra dimension introduced by the volume compared to a standard 2D environment texture.

Most of the techniques on rendering volumes have been focused on scattering effects in volumes, using photon maps [Jensen and Christensen 1998], and more recently with photon beams [Novák et al. 2012] or improved sampling techniques [Kulla and Fajardo 2012] (see Cerezo's survey [2005] for an overview). We focus on the emission effect from a volume, where the receiver can be either a volume or a surface. By completely ignoring the scattering, we are only handling the illumination from the volume and not the rendering of the volume itself.

Zhang et al. [2011] proposed a cluster-based method to importance sample volumetric flames; our approach relies solely on MIS, so the volumetric light is seen as any other light by the renderer and requires no special treatment or preprocessing.

In Section 2, we present the simplified volume rendering equation which is the basis of our algorithm. In Section 3.1, we discuss how light and BRDF sampling can be handled in a homogeneous volume, and in Section 3.2 how to extend those results to heterogeneous volumes. Finally, we give a few implementation details in Section 4 and conclude in Section 5 showing our results.

2. Volumetric Rendering Equation

The radiative transport equation [Kajiya and Von Herzen 1984] [Glassner 1995] [Kniss et al. 2003] defines how light travels and is emitted in participating medium[1]:

$$(\omega \cdot \nabla)L(x,\omega) + \sigma_{\mathrm{T}}(x)L(x,\omega) = \varepsilon(x,\omega) + \sigma_{\mathrm{S}}(x)\int_{\Omega} p(x,\omega,\omega')L(x,\omega') \;\; d\omega'.$$

Table 1 describes the variables that appear in this and our other equations. We now simplify this general equation by considering only a fixed ray direction, ω_0.

[1]Following Glassner, $(\omega \cdot \nabla)L(\vec{x},\omega) = x_{\omega}\frac{dL(\vec{x},\omega)}{dx} + y_{\omega}\frac{dL(\vec{x},\omega)}{dy} + z_{\omega}\frac{dL(\vec{x},\omega)}{dz}$

Symbol	**Description [Units]**
x	A point [m]
t	A distance [m]
ω	Unit direction vector away from a surface point x [1]
θ	angle between the surface normal N and ω_i (or segment $[x;x']$) [r]
s	a line segment that lies within a medium
$\|s\|$	the length of s
$L(x,x')$	radiance from x to x' [$\mathrm{Wm^{-2}sr^{-1}}$]
$L(x,\omega)$	radiance from x in the ω direction [$\mathrm{Wm^{-2}sr^{-1}}$]
$f(x,x',x'')$	surface BRDF at x' from x to x'' [$\mathrm{sr^{-1}}$]
$f(x,\omega_\mathrm{o},\omega_\mathrm{i})$	surface BRDF at x from direction ω_i to direction ω_o [$\mathrm{sr^{-1}}$]
p	probability distribution function
$p(t \mid \omega)$	differential probability of scattering at distance t when propagating in direction ω [1]
$p(t)$	differential probability of scattering at distance t independent of direction ($p(t) = \int_\Omega p(t \mid \omega)d\omega$) [1]
$\tau(x,x')$	volumetric attenuation between x and x'
$\tau(x,\omega,t)$	volumetric attenuation between x and $x+\omega t$
$\delta_\mathrm{vis}(x,x')$	geometric visibility between x and x'
$\delta_\mathrm{vis}(x,\omega,t)$	geometric visibility between x and $x+\omega t$
ε	total volumetric emission, or uniform emission [$\mathrm{Wm^{-3}sr^{-1}}$]
ε_i	i^th voxel volumetric emission [$\mathrm{Wm^{-3}sr^{-1}}$]
V	total volume [$\mathrm{m^3}$]
V_i	i^th voxel volume [$\mathrm{m^3}$]
σ_T	transmission coefficient [$\mathrm{m^{-1}}$]
σ_S	scattering coefficient [$\mathrm{m^{-1}}$]
σ_A	absorption coefficient [$\mathrm{m^{-1}}$]

Table 1. Notation used in this paper.

The change in radiance over an infinitesimal segment in direction ω_0 is the sum of the emission of that segment and the scattering from all incoming directions:

$$L'(t)+\sigma_\mathrm{T}(t)L(t) = \varepsilon(t)+\sigma_\mathrm{S}(t)\int_\Omega p(t,\omega')\, L(t,\omega')d\omega'.$$

This is obviously still too difficult to solve analytically, so we simplify further by keeping only what interests us in the case of a volumetric light, that is, absorption, σ_A, and emission, ε. We completely ignore scattering; in other words, we are setting σ_S to 0, thus the extinction $\sigma_\mathrm{T} = \sigma_\mathrm{S}+\sigma_\mathrm{A}$ simplifies to the absorption [Max 1995] and

$$L'(t)+\sigma_\mathrm{A}(t)L(t) = \varepsilon(t).$$

This is a standard first-order linear equation, and solving for the lighting in our given ray direction (Figure 2), we obtain

$$L(t) = L(x_0,\omega_0)\exp\left(-\int_0^t \sigma_\mathrm{A}(t')dt'\right)+\int_0^t \varepsilon(t')\exp\left(-\int_{t'}^t \sigma_\mathrm{A}(t'')dt''\right)dt'.$$

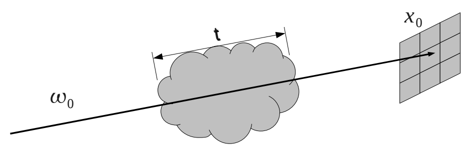

Figure 2. Volume integration: the radiance along a ray is the sum of the media radiance (segment t in the volume) and the radiance at the nearest surface point, x_0.

Within an isotropic, homogeneous volume, the emission, ε, and density, σ_A, are constant. Under these conditions, and when $\sigma_A > 0$, the equation becomes

$$L(t) = \frac{\varepsilon}{\sigma_A}(1 - e^{-\sigma_A t}) + L(x_0, \omega_0)e^{-\sigma_A t}. \tag{1}$$

Another equation we will use for light sampling is the special case of attenuation only, where $\varepsilon(t)$ is zero:

$$L'(t) + \sigma_A(t)L(t) = 0.$$

The solution is

$$L(t) = L(x_0, \omega_0)\ \exp\left(\int_0^t -\sigma_A(t')dt'\right).$$

and for a homogeneous volume,

$$L(t) = L(x_0, \omega_0)\ e^{-\sigma_A t}. \tag{2}$$

Now that we have simplified our transport equation and have solved for emission in a given direction, we can derive our estimators to use in importance sampling.

3. Emissive Volume Sampling and Evaluation

The results above can be applied directly to the case of a homogeneous volume. We will show how to sample in practice in the next section. Then, we will make use of this when generalizing to heterogeneous media in Section 3.2.

3.1. Homogeneous Emitting Volume

An isotropic homogeneous emitting volume has a total emitting intensity of εV. Since everything is constant, for the light sampling we can use a uniform point sampling of the volume. For the BRDF sampling, there is an extra step required. As opposed to a standard area light evaluation, volume BRDF sampling will resolve into a segment. We still need to compute the total intensity of that segment by estimating the line integral over it.

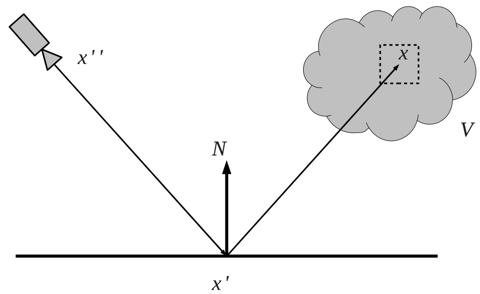

Figure 3. Light sampling in a homogeneous medium: we pick a random point x in the volume, V, and we estimate its contribution at x' towards the viewer x''

3.1.1. Light Sampling

The light sampling is relatively straightforward. We need to sample a point inside the volume and then compute the absorption due to the density of the volume from this point (Figure 3).

Changing the integration domain to $\mathbb{R}^3$ and using the three-point form of the rendering equation [Raab et al. 2007], at a shading point on a surface, the contribution of an emissive volume is

$$L(x',x'') = \int_V f(x,x',x'')\cos(\theta)G(x,x')\tau(x,x')\varepsilon dV(x).$$

with $G(x,x') = \frac{\delta_{\text{vis}}(x,x')}{\|x-x'\|^2}$

Since we are taking a sample randomly in the volume, $p(x) \propto \frac{1}{V}$, the resulting estimate is

$$L(x',x'') \approx \frac{f(x,x',x'')\cos(\theta)G(x,x')\tau(x,x')\varepsilon}{p(x)}.$$

The attenuation $\tau(x,x')$ is given by the right exponential of Equation (2) for a homogeneous medium; in this case t is the distance traveled in the volume on segment $[x;x']$. For non-convex volumes, or volumes with "holes," there will be multiple segments. These will be handled by the algorithm for heterogeneous volumes in Section 3.2.

3.1.2. BRDF Sampling

The sampling direction is chosen according to the BRDF receiving light; then, in the emissive volume, we still need to estimate a line integral, i.e., from the input direction, we need to provide a point, emission value and probability distribution function (PDF) value. The first step is to compute the intersection between the line and the volume to get the segment range. Once we have this segment, we sample a point on it (Figure 4).

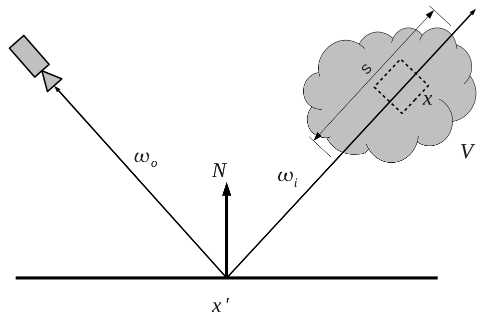

Figure 4. BRDF sampling in a homogeneous medium: from the BRDF at x' and based on the viewing direction, ω_o, we first pick a direction, ω_i, leading to a segment, s, in the volume, V, then a point, x, on the segment.

$$L(x',\omega_o) = \int_\Omega f(x',\omega_i,\omega_o)\cos(\theta)L(x',\omega_i)d\omega_i$$

with

$$L(x',\omega_i) = \int_s \delta_{vis}(x',\omega_i,t)\tau(x',\omega_i,t)\varepsilon dt.$$

Segment s is the portion of the ray that lies within the medium, as shown in Figure 4.

We saw in Section 2 that we can analytically compute the emission for a given direction in a homogeneous volume. We will use this function directly as a PDF, the corresponding cumulative distribution function (CDF) is given by Equation (1) and is easily invertible. Using the usual conditional probability notation, the differential probability of picking t given a direction ω_i is

$$p(t \mid \omega_i) \propto \tau(x',\omega_i,t)\varepsilon. \tag{3}$$

(For any random number, χ, the CDF inversion gives $t = -\frac{1}{\sigma_A}\ln(1-(1-e^{-\sigma_A|s|})\chi)$.)

Using $p(\omega_i)$ to denote the PDF resulting from the BRDF sampling, the resulting estimate can be written as

$$L(x',\omega_o) \approx \frac{f(x,\omega_i,\omega_o)\cos(\theta)\delta_{vis}(x',\omega_i,t)\tau(x',\omega_i,t)\varepsilon}{p(\omega_i)p(t \mid \omega_i)}$$

which reduces to

$$L(x',\omega_o) \approx \frac{f(x,\omega_i,\omega_o)\cos(\theta)\delta_{vis}(x',\omega_i,t)L_e(x',\omega_i)}{p(\omega_i)} \tag{4}$$

since Equation (3) is $p(t \mid \omega_\mathrm{i}) = \frac{\tau(x',\omega_\mathrm{i},t)\varepsilon}{L_\mathrm{e}(x',\omega_\mathrm{i})}$, where $L_\mathrm{e}(x',\omega_\mathrm{i})$ is the normalization factor:

$$\begin{aligned} L_e(x',\omega_\mathrm{i}) &= \int_S \tau(x',\omega_\mathrm{i},t)\varepsilon dt \\ &= \frac{\varepsilon}{\sigma_\mathrm{A}}(1-e^{-\sigma_\mathrm{A}|s|}). \end{aligned}$$

This gives us a nearly optimal sampling; the only part not accounted for is the visibility function, $\delta_{\mathrm{vis}}(x',\omega_\mathrm{i},t)$. Next, we derive the sampling for the more complex heterogeneous case.

3.2. Heterogeneous Emitting Volume

Now, let's consider that a heterogeneous volume is represented as a collection of a finite number of homogeneous sub-volumes. For a grid-type volume, each voxel is going to be a cubic homogeneous isotropic emitting volume. By initially sampling the set of voxels, we can utilize our homogeneous sampling strategies for heterogeneous volumes.

3.2.1. Light Sampling

For the light sampling, we sample the voxels based on the distribution of ε. Since we have a finite number of voxels, we can use the same method used in Pharr's infinite area light source sampling [2004] for the environment map sampling, replacing the pixel colors by ε and adding one more dimension. First, we choose a plane by sampling from the distribution where we have marginalized the second and the third dimension. Then, we choose a line from the plane using the distribution for the plane marginalized over the third dimension. Finally, we choose a voxel from the distribution of the chosen line (Figure 5).

Once we have a voxel, we just need to uniformly sample a point inside it, which is exactly the process described in the previous section. For the following section, we will consider that we have a grid, η, with total emission $\varepsilon = \sum_{i\in\eta}\varepsilon_\mathrm{i}$:

$$L(x',x'') = \sum_{i\in\eta}\int_{V_i} f(x,x',x')\cos(\theta)G(x,x')\tau_\eta(x,x')\varepsilon_i dV_i(x). \tag{5}$$

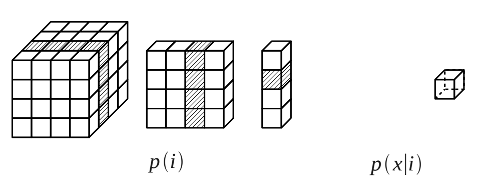

Figure 5. Light grid: first, we choose which voxel to sample, then we choose a point within that voxel.

Also note that τ_η is not only the attenuation within the voxel, i, but the full attenuation due to all the voxels between the shading point and the sampled point. The exponential function representing the attenuation is now a piecewise exponential.

By taking a voxel based on a discrete PDF, $p(i) \propto \varepsilon_i$, and then a point inside this voxel using $p(x \mid i) \propto \frac{1}{V_i}$, the resulting final estimate is

$$L(x', x'') \approx \frac{f(x, x', x'')\cos(\theta)G(x, x')\tau_\eta(x, x')\varepsilon_i}{p(i)p(x \mid i)}.$$

Again, the attenuation is given by Equation (2) for all the voxels between the shaded point and the sampled point.

3.2.2. BRDF Sampling

Similar to the homogeneous volume case, we can use the emission function directly as a PDF. The difference is that, as we did with the attenuation function, we have a piecewise-exponential function instead of a pure exponential function. The piecewise-exponential function is constructed by computing all voxels that the ray will intersect, from front to back. To perform the CDF inversion from this piecewise-exponential function, we must first determine the containing voxel and then use the algorithm from Section 3.1.2 to choose a point within this voxel (see Figure 6):

$$L(x', \omega_o) = \int_\Omega f(x', \omega_i', \omega_o)\cos(\theta)L(x', \omega_i)d\omega_i$$

with

$$\begin{aligned} L(x', \omega_i) &= \int_S \delta_{\mathrm{vis}}(x', \omega_i, t)\tau_\eta(x', \omega_i, t)\varepsilon(t)dt \\ &= \sum_{i \in S}\int_{s_i} \delta_{vis}(x', \omega_i, t)\tau_\eta(x', \omega_i, t)\varepsilon_i(t)dt. \end{aligned}$$

The resulting final estimate is

$$L(x', \omega_o) \approx \frac{f(x, \omega_i, \omega_o)\cos(\theta)\delta_{\mathrm{vis}}(x', \omega_i, t)\tau_\eta(x', \omega_i, t)\varepsilon_i}{p(\omega_i)p(t \mid \omega_i)} \tag{6}$$

which simplifies again to Equation (4).

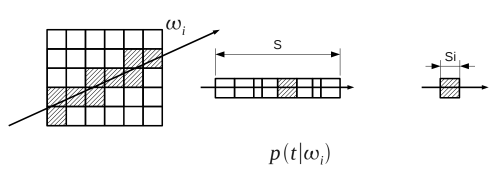

Figure 6. BRDF grid: we find the intersecting voxel group and then choose one within the group.

4. Implementation Details

Combining these two strategies with MIS is straightforward [Veach and Guibas 1995], but the implementation in a ray-tracer can be tricky. Usually the light sampling is done by light shaders, and the BRDF sampling is done by the surface shader. This makes it difficult (or impossible) for the light shader to modify the PDFs produced by the surface.

With a power heuristic with an exponent of 2 for the MIS weight, the light-sampling estimation is (a few parameters have been omitted and notation simplified for brevity):

$$L_{\text{light}} = \frac{p(x)^2}{[p(\omega)p(t \mid \omega)]^2 + p(x)^2} \cdot \frac{f() \cdot L}{p(x)}.$$

The$p(\omega)$ and $f()$ values are returned by the BRDF shader. The $p(x)$ and L values are returned by the light shader, along with an additional value of $p(t \mid \omega)$ that needs to be applied on $p(\omega)$.

An optimization we have used for this problem is to return pre-divided values in our light shader. By defining the light shader to return $p(x)/p(t \mid \omega)$, $L/p(t \mid \omega)$, we can avoid passing the third parameter:

$$\begin{aligned} L_{\text{light}} &= \frac{[p(x)/p(t \mid \omega)]^2}{p(\omega)^2 + [p(x)/p(t \mid \omega)]^2} \cdot \frac{f() \cdot L/p(t \mid \omega)}{p(x)/p(t \mid \omega)} \\ &= \frac{p(x)^2}{[p(\omega)p(t \mid \omega)]^2 + p(x)^2} \cdot \frac{f() \cdot L}{p(x)}. \end{aligned}$$

We can apply the same method to the BRDF sampling:

$$\begin{aligned} L_{\text{brdf}} &= \frac{p(\omega)^2}{p(\omega)^2 + [p(x)/p(t \mid \omega)]^2} \cdot \frac{f() \cdot L/p(t \mid \omega)}{p(\omega)} \\ &= \frac{[p(\omega)p(t \mid \omega)]^2}{[p(\omega)p(t \mid \omega)]^2 + p(x)^2} \cdot \frac{f() \cdot L}{p(\omega)p(t \mid \omega)}. \end{aligned}$$

5. Results and Discussion

A reflection of a flame on a plane with various roughness values is shown in Figure 7. The light-sampling contribution is dominant when the surface is rough (or the light source is small), and the BRDF sampling contribution becomes greater when the surface becomes a mirror (or if the light is large). Classically, thanks to MIS, we can maintain low variance for all intermediate configurations.

Figure 8 shows a comparison between our algorithm and the point-based method. Although the point-based method gives similar results for diffuse surfaces like the walls, on specular objects, the individual points become apparent. Again, our method gives very high quality results on all surfaces and requires less set-up by the artist.

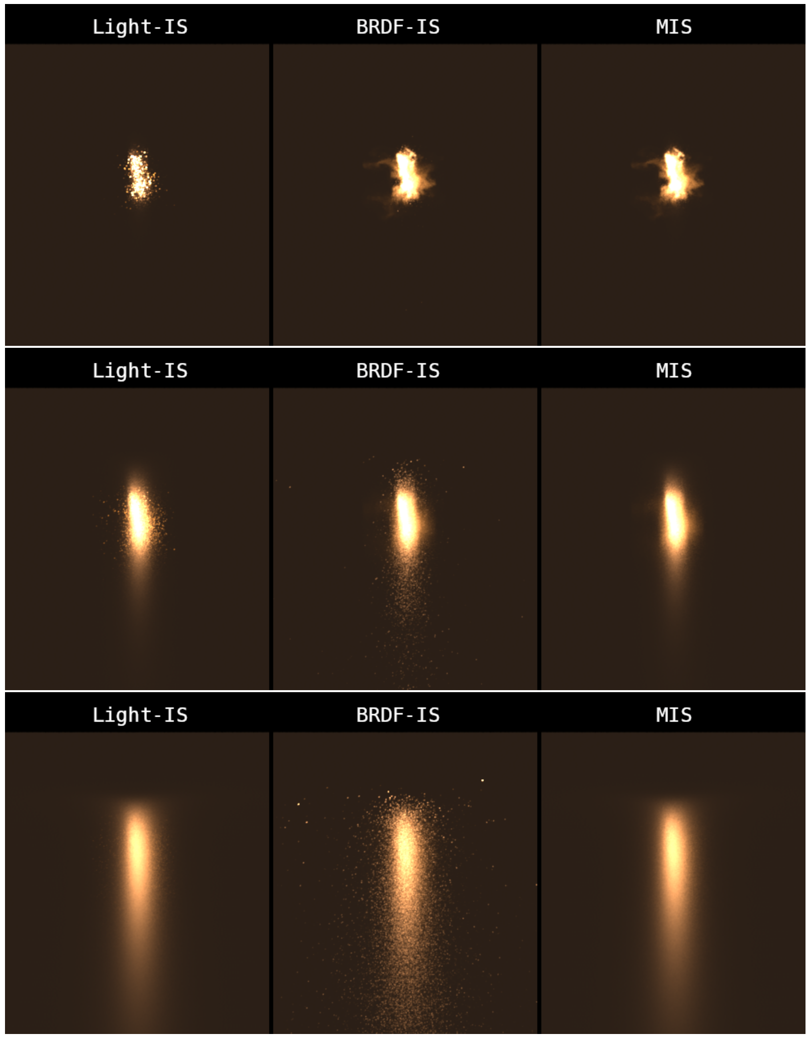

Figure 7. Flame reflection on a glossy plane with varying roughness.

This leads to the main advantage of using our method in production. Visually, we could achieve a close-up using multiple point lights, however the advantages of using the flames directly are:

- the flickering of the flames will directly show up as an animated illumination (no need to try to fake it with a noise on the intensity of the light);
- if the flame changes in the process of building the scene, there is no need to redo the lighting;
- if the scene contains specular objects, they will reflect the flame and not a bunch of point lights.

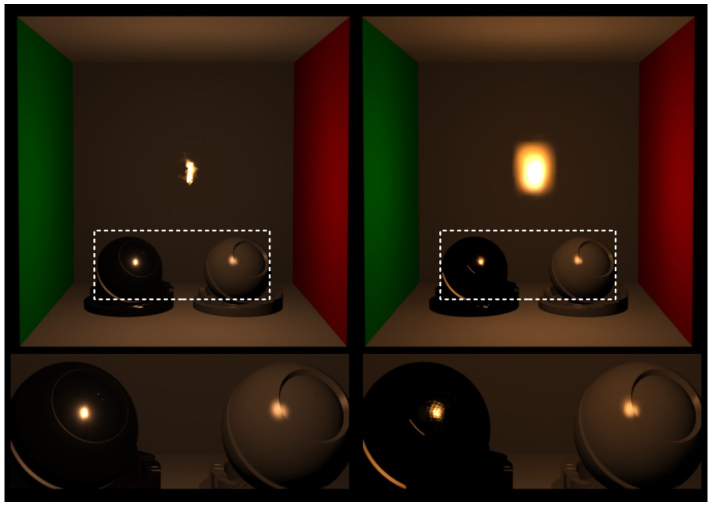

Figure 8. Our method (left) maintains high-quality lighting even on specular surfaces where artifacts are visible with the volumetric point-based method (right). Both images were rendered without occlusion and GI. A large amount of manual work was necessary to get a visual match using the point-based method, whereas our method gives us the right result automatically.

While MIS makes this algorithm very robust, there are still cases where sampling can be problematic. If the volume is very dense, the light sampling can become less efficient by sampling voxels that have high emission but which do not contribute much to the result because of a high extinction between it and the shaded point. One solution would be to build directional CDFs. This would also help in the case of anisotropic emission, but depending on the size of the volume, memory cost could be a problem. In practice, we didn't find such cases.

One other point left unresolved by our method is motion-blur. The sampling itself is ignoring time. We could use an already blurred volume data, if the need arose; this approximation should be good enough in most cases.

6. Conclusion

We were able to provide two sampling strategies for a volumetric light based on heterogeneous voxel data:

- light sampling based on the voxel emission distribution (Equation (5))
- BRDF sampling based on the product of transmittance and emission along a line (Equation (6))

Since those two strategies are sampling the same domain, MIS is fully functional. The algorithm is robust and can handle a large variety of surfaces materials (diffuse to mirror) and volumes without any special cases, as shown in Figure 9.

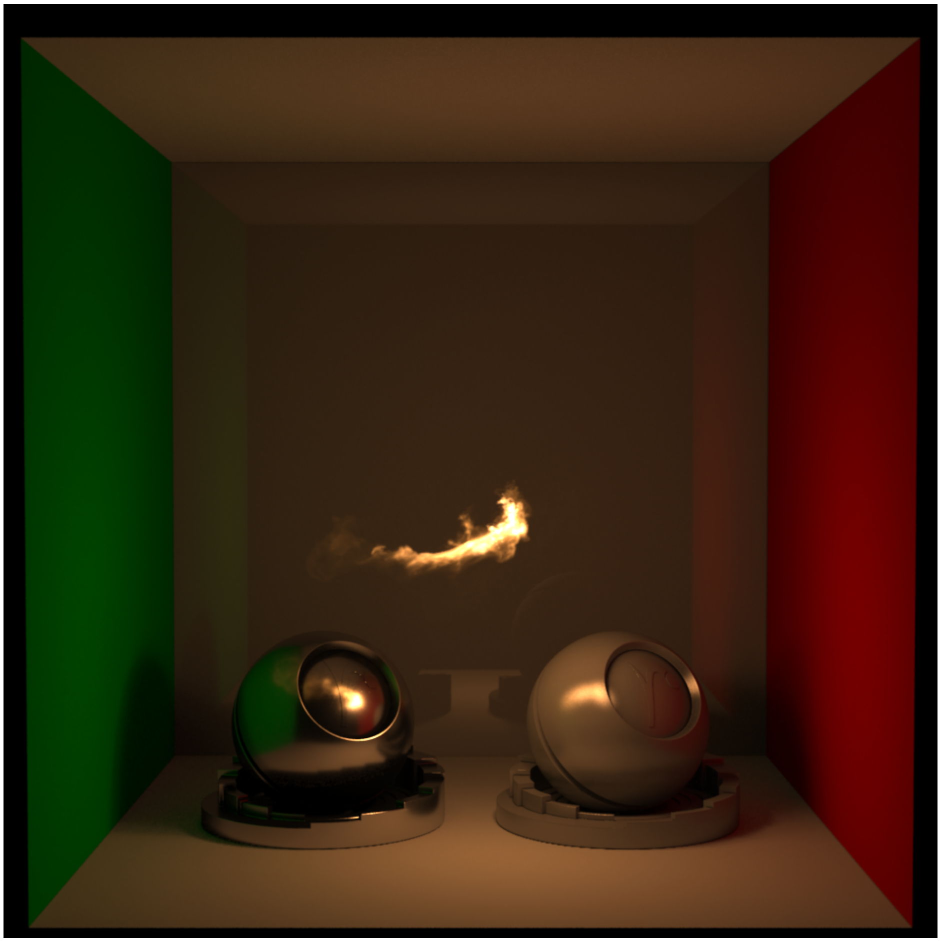

Figure 9. Cornell box with specular and diffuse objects. This scene with full global illumination was rendered at 1K resolution in 40 seconds on 10 cores with 64 light samples and 64 BRDF samples per shading point.

Our method greatly simplifies workflows and immediately plugs into any ray-tracer with sampled light capability. An additional advantage is that since this method solely relies on importance sampling and does not require any preprocessing or extra steps, it will also integrate naturally in a progressive renderer.

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Villemin and Hery, Practical Illumination from Flames, *Journal of Computer Graphics Techniques (JCGT)*, vol. 2, no. 2, 142–155, 2013
http://jcgt.org/published/0002/02/10/

Received:	2013-07-12		
Recommended:	2013-11-01	Corresponding Editor:	Jaakko Lehtinen
Published:	2013-12-31	Editor-in-Chief:	Morgan McGuire